

MA114 Summer II 2018  
Worksheet 2b - Partial Fractions  
6/11/18

Solutions.

1. Write out the general form of the partial fraction decomposition but do not solve for the coefficients.

$$\text{a) } \frac{1}{x^2 + 3x + 2} = \frac{1}{(x+1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2} \quad (\text{linear factors, not repeated})$$

$$\text{b) } \frac{2x+1}{x^2+4x+4} = \frac{2x+1}{(x+2)^2} = \frac{A}{x+2} + \frac{B}{(x+2)^2} \quad (\text{linear factor, repeated})$$

$$\text{c) } \frac{x}{(x^2+1)(x+1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2} + \frac{Cx+D}{x^2+1} \quad (\text{two distinct linear factors \& one irreducible quadratic})$$

2. Compute the following integrals.

$$\text{a) } \int \frac{x-9}{(x+5)(x-2)} dx,$$

$$\begin{aligned} &= \int \left( \frac{2}{x+5} - \frac{1}{x-2} \right) dx \\ &= \frac{2 \ln|x+5| - \ln|x-2| + C}{1} \\ &= \underline{\underline{\ln \left| \frac{(x+5)^2}{x-2} \right| + C}} \end{aligned}$$

$$\frac{x-9}{(x+5)(x-2)} = \frac{A}{x+5} + \frac{B}{x-2} \quad \begin{array}{l} \text{clear denominators} \\ \downarrow \\ \Rightarrow x-9 = A(x-2) + B(x+5) \end{array}$$

$$\text{Let } x=2: -7 = 7B$$

$$\underline{B=-1}$$

$$x=-5: -14 = -7A$$

$$\underline{A=2}$$

$$b) \int \frac{10}{(x-1)(x^2+9)} dx.$$

$$\frac{10}{(x-1)(x^2+9)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+9} \Rightarrow 10 = A(x^2+9) + Bx(x-1) + C(x-1)$$

$$\text{Let } x=1: 10 = A(10), \text{ so } \underline{A=1}$$

$$x=0: 10 = 9 + C(-1)$$

$$\underline{C=-1}$$

$$x=2: 10 = 13 + 2B - 1$$

$$-2 = 2B$$

$$\underline{B=-1}$$

$$= \int \left( \frac{1}{x-1} - \frac{x+1}{x^2+9} \right) dx$$

$$= \int \left( \frac{1}{x-1} - \frac{x}{x^2+9} - \frac{1}{x^2+9} \right) dx$$

$$= \underline{\ln|x-1| - \frac{1}{2} \ln|x^2+9| - \frac{1}{3} \arctan\left(\frac{x}{3}\right) + C}$$

3. Compute

$$\int \frac{1}{\sqrt{x} - \sqrt[3]{x}} dx$$

by first making the substitution  $u = \sqrt[6]{x}$ .

$$u = x^{1/6}, \quad du = \frac{1}{6} x^{-5/6} dx$$

$$dx = 6x^{5/6} du$$

$$= 6u^5 du$$

$$\int \frac{1}{\sqrt{x} - \sqrt[3]{x}} dx = \int \frac{1}{x^{3/6} - x^{2/6}} dx = \int \frac{1}{u^3 - u^2} \cdot 6u^5 du$$

$$= 6 \int \frac{6u^3}{u-1} du$$

$$= 6 \int u^2 + u + \frac{1}{u-1} du$$

$$= 6 \left[ \frac{1}{3}u^3 + \frac{1}{2}u^2 + u + \ln|u-1| \right] + C$$

$$= \underline{2\sqrt{x} + 3\sqrt[3]{x} + 6\sqrt{x} + 6 \ln|\sqrt[6]{x}-1| + C}$$